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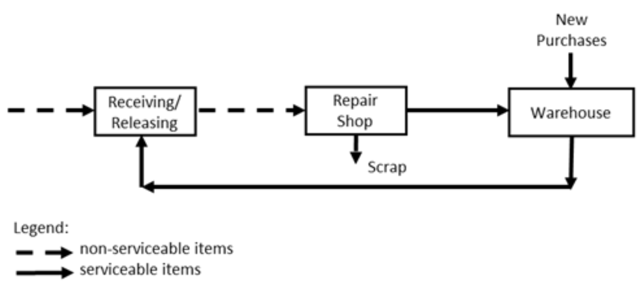
Controlling repairable inventories of critical items

Marilyn C. Mabini

Research Center for the Natural and Applied Sciences, University of Santo Tomas, España Blvd., Manila

Graphical Abstract



Abstract

The present paper is concerned with the problem of controlling repairable inventories of critical items, so that these items are available when they are needed. The context used is that of the rail transport, or train system. We present a nonlinear integer programming formulation for a single-level repairable inventory system. The formulation strives to minimize the sum of the expected annual costs of carrying inventories and of operational delay due to item unavailability on demand. We apply a constraint limiting the allowable probability of item availability to a pre-set minimum value. Using hypothetical data, we solve the model using a spreadsheet-based solver developed for this purpose. The formulation, taken for each item, is convex. Our model promises to be a useful tool for repairable-item inventory control in the railway industry, or in the transportation industry in general.

Keywords: repairable inventories; mathematical programming

Corresponding author: mcmabini@ust.edu.ph
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Introduction

Repairable items are important in industries where systems employed consist of costly parts which, upon failure, can be restored to serviceable state through repair. One such industry is the railway industry. A train can go out of service for an extended period, when any of its critical parts fails. Railroad organizations that can afford it tend to overstock these items to avoid emergency situations. With its cost implications, overstocking is certainly not sound practice. Understocking cannot be justified either, because parts availability is vital to get the train back in operation soonest.

The present paper is concerned with the problem of controlling repairable inventories of critical items, specifically using the context of the train system, for modeling and illustrative purposes.

We propose a nonlinear integer programming formulation for a single level multi-item repairable inventory system. The formulation seeks to minimize the expected annual cost of carrying inventories and of incurring delays in operations due to unavailability of items at the time that they are needed. The model includes a service constraint which sets a minimum on the probability that an item is available when needed. It is a non-indentured item version of the model by Mabini and Christer [1].

We solve the model using hypothetical data. Originally, our intention was to use a Philippines-based organization. The same could not be pursued, however, because of problems related to data accessibility. Nevertheless, the present model is essentially based on the general problem environment in the Philippine railway industry. As such, the model would be helpful to controlling repairable stocks of critical train parts in a single level inventory system.

For the purposes of this paper, the terms “part” and “item” shall be used interchangeably.

Problem Environment. We study the single echelon inventory control system for repairable items supporting train operations. The coaches that comprise a train have parts, which are subject to failure. The present study is concerned with critical train parts which are repairable, or are capable of being restored back to serviceable state upon repair. These are typically low demand and costly items.

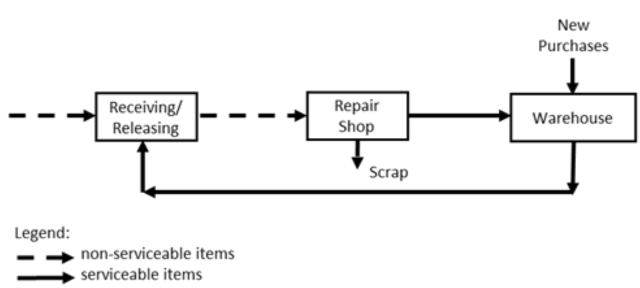

Figure 1. Flow of items in the system.

When a critical part fails, train operation is interrupted. The affected coach is brought to the depot, the failed part is pulled out and sent to the repair facility located within the depot. Meanwhile, a replacement part, if available, is drawn from stocks and installed in the coach. If such part is not available, it is requisitioned, and the coach remains non-operational until the replacement part becomes available either from repair or from a fulfilled purchase order.

The warehouse, which is located in the depot, stores serviceable items, repaired or acquired from a new purchase, and non-serviceable items due for repair. Non-repairable items are scrapped.

It is desired to determine the inventory levels of critical parts, which will minimize cost, while satisfying a specified service level. The latter may be represented by a minimum probability at which the availability of the needed parts is guaranteed.

Related Literature. Typically, a single-echelon repairable inventory system has a repair facility and a warehouse where serviceable and non-serviceable items awaiting repair are kept. Here, serviceable items come from two sources: the repair shop turning out recovered items, and external suppliers responding to purchase orders.

De Haas [2] studies a system consisting of a group of repair departments and stock locations designed to provide serviceable engines to aircraft. He emphasizes the need to coordinate the performance of the repair departments and the stock locations, and to set service level targets for both.

Teunter [3] proposes an EOQ model for an inventory system of items that can be repaired, refurbished, or remanufactured. The model, which allows disposal of nonrecoverable items, differentiates holding costs for recovered and manufactured items. A variant of this model is found in Koh, et al [4], whose work provides a joint EOQ and EPQ formulation for situations where stationary demand can be satisfied from repaired and newly purchased items.

Konstantaras [5] extends the work of Koh, et al [4] in his model for finding the optimal inventory level of recoverable items that would signal the inspection and recovery processes, and determine the economic order quantity for procurement.

In Alfredsson [6], we find a support system for a fleet of technical systems, and a mathematical framework for finding the quantity of spares and test equipment needed and for determining where repair should take place. The author proposes an algorithm to find cost-efficient configurations for the support system.

Wong, et al [7] propose a multi-item, continuous review model for a repairable spare parts inventory system supporting expensive technical systems with high target availability levels. The system allows lateral and emergency shipments to fill the demand in stockout situations.

In his study of a repairable item system for expensive parts of machines, Tracht [8] investigates the impact of varying repair capacity on the system. He shows how the stock levels should be adjusted so that a maximum backorder level of waiting request is guaranteed for the entire year.

In another paper, Teunter [9] derives formulas for determining optimal lot sizes for the production or purchase of new items and for the recovery of returned items. His cost minimization base formulas are valid for finite and infinite production and recovery rates.

The possibility of complete pooling of stocks of repairable items among multiple companies is analyzed by Wong in two papers with his collaborators [10, 11]. In the former paper, Wong, et al present a model which minimizes the sum of holding, downtime, and transshipment costs. In the latter paper, Wong, et al propose a multi-dimensional Markovian model for the problem with delayed lateral transshipments.

The multi-period single-item repairable inventory system with stochastic new and warranty demands is investigated by Lin, et al [12]. The authors assume that newly-procured items and repaired items from repairable warranty returns are non-distinguishable, and that new demand has a higher priority than warranty demand within a period. A dynamic programming model is proposed for the problem. From the results, the authors point out that the optimal policy is completely defined by three period-dependent thresholds: the purchase-up-to level, the repair-up-to level, and the scrap-down-to level.

Turan, et al [13] study the design problem of a single repair shop supporting a repairable multi-item spare part supply system. Stochastic item failure and repair time are assumed. A sequential solution heuristic is proposed to solve the joint problem of resource pooling, inventory allocation, and capacity level designation of the repair shop. Their approach involved the decomposition of the repair shop in sub-systems by clustering, which allowed the use of queue theoretical approximations to optimize the inventory and capacity levels. The approach was found to result in 10% to 30% cost reductions.

Materials and methods

To address the problem of provisioning repairable critical parts to support train operations, we develop an integer nonlinear programming model whose objective is to minimize the cost of carrying inventories and of delays due to interruption in operations arising from the unavailability of parts to replace the failed ones. Incorporated in the model is a service level constraint meant to ensure that part availability does not go below a specified probability. The model is designed to determine the inventory levels of items that would achieve the said objective while satisfying the service level constraint.

We solve the model by means of an integer search for the optimal inventory levels using spreadsheet computations. This process is fairly easy to implement because the search range is relatively limited, owing to the fact that the items involved are low demand items.

Formulating the Model

Assumptions. We lay down the following assumptions underlying our mathematical model for the provisioning of critical train parts.

1. Demand for critical train parts follows a Poisson process.

2. The average demand rate for each critical item is known and fixed.

3. Demands not immediately met are backordered, with corresponding costs due to delays in train operation.

4. For a given part, the probability of failure is independent of failures occurring for other parts.

5. The probability of a part being repairable and its probability of being non-repairable are known and fixed.

6. There is ample capacity in the repair facility for critical item repair to start immediately in the event of failure.

7. The inventory holding cost is the same for both serviceable and non-serviceable items.

8. Estimates of model parameters are known and fixed: unit holding and operational delay costs per unit time, unit prices of the items, purchase lead times, repair times, repair output rates.

9. Items that are non-repairable are scrapped and replenished through purchases.

Notation. We define the notation used in the model as follows.

λ_i average demand rate for item i (units/day)

μ_i average number of units of item i repaired per day

C_{del} penalty cost per non-operational train coach per day, \$/coach-day

Di expected additional delay (days) in replacing item i in a train coach due to item i stockout

HC total expected annual holding cost of serviceable and non-serviceable items in the system, \$/year

H_i unit inventory holding cost per unit time for item i, \$/unit - year

N total number of critical items

p(x;z) Poisson probability that x units are demanded within a given period when the average demand within the same period is z

PC total expected annual purchase cost of parts, \$/year

P_i purchase price of item i, \$/unit

RC total expected annual repair cost of items, \$/year

RC_i repair cost for item i, \$/unit

r_i probability that item i is repairable

R_i total repair time (days) for item i including transportation time to the repair shop before repair and to the warehouse after repair

r_{ni} probability that item i is not repairable

R_{pi} total time (days) to remove item i from a train coach, obtain a replacement from the warehouse and replace the same in the coach

SC expected annual delay cost, \$/year

S_i stock level of item i

TC total expected annual cost of operating the inventory system, \$/year

T_i expected resupply time (days) for part i from the repair shop and from the supplier

t_i expected total time of operational delay (days) due to failure of item i

T_{pi} purchase lead time (days) for item i

Mathematical Formulation. We present the mathematical formulation for the problem below.

$$\underset{(S_i, S_{ij})}{\text{Minimize}}\ TC = HC + PC + RC + SC \quad (1)$$

subject to:

$$p(\text{item i available on demand}) \geq = 0.90, \qquad \text{for all i} \quad (2)$$

where,

$$HC = \sum_{i=1}^{N} H_i[\sum_{x=0}^{Si-1}(S_i - x)p(x;\lambda_i T_i) + (1-r_i)\ \lambda_i T_i] \quad (3)$$

$$PC = 365\sum_{i=1}^{N} \lambda_i r_{ni} P_i \quad (4)$$

$$RC = 365\sum_{i=1}^{N} \mu_i C_i \quad (5)$$

$$SC = 365\sum_{i=1}^{N} C_{del} t_i \lambda_i \quad (6)$$

$$p(\text{part i available on demand}) = \sum_{x=0}^{S_i} \frac{(\lambda_i T_i)^x e^{-\lambda_i T_i}}{x!} \quad (7)$$

The decision variables S_i are nonnegative integers for all i.

The objective function in equation (1) strives to minimize the sum of the expected annual inventory holding, purchasing, repair, and operational delay costs. The holding cost, HC in equation (3), is the expected total annual carrying cost of serviceable and non-serviceable inventories.

In equations (4) and (5), respectively, we find that the purchase and repair costs, PC and RC, are independent of the stock levels, S_i. For this reason, PC and RC will not affect the optimization process to determine the desired stock levels; therefore, these costs will be excluded from the process.

The delay cost SC in equation (6), is a function of item backorders. We have the relationship below with respect to t, the expected number of operational delay days due to part failure. For part i, the following will apply

$$t_i = R_{pi} + D_i \tag{8}$$

where D_i, the expected number of days delay due to a stockout on item i, is given by the expected number of item i backorders divided by the daily demand rate for the same part [14]. That is,

$$D_i = \frac{1}{\lambda_i} \sum_{x>S_i} (x - S_i) p(x; \lambda_i T_i). \tag{9}$$

The expected resupply time, T_i, for item i is the expected time for the item to be repaired (including transportation time to and from the repair facility), or to arrive from a purchase order, in the event of scrapping. This is given below,

$$T_i = r_i R_i + r_{ni} T_{pi} \tag{10}$$

where,

$$r_i + r_{ni} = 1$$

In (2), we apply a constraint to limit the probability of parts availability to a minimum level, say, 0.90.

Illustrative Problem and Results

We solved the model using a sample problem with three items, whose characteristics are given in Table 1. Our data are hypothetical, but chosen such that demand rates and unit prices vary from the least frequently demanded and least costly Item 1 to the most frequently demanded and most expensive Item 3.

Table 1. Data for the illustrative problem.

Item	Demand Units/day	Unit Price, $	r_i	r_{ni}	T_{pi} days	R_i days	R_{pi} days
1	0,0247	6300	0.90	0.10	120	4	1
2	0.0548	8500	0.90	0.10	90	4	1
3	0.1041	60000	0.90	0.10	150	5	1.5

We assign an annual inventory holding cost per unit equivalent to 25% of the item price, and a penalty cost, C_{del} = \$2000/coach-day.

To explore how the optimal stock levels will behave with variations in item repairability and in the cost of delay due to item unavailability, we solved four instances of the problem. The problem instances are described in Table 2.

Table 2. Problem instances solved using the model.

Problem Instance	Description
1	based on the data, Table 1, and C_{del}*=\$2000
2	based on the data, Table 1, and C_{del} =\$20000
3	based on the data, Table 1, r_i = 0.75, and C_{del}=\$2000
4	based on the data, Table 1, r_i = 0.75,

*C_{del} is the penalty cost per non-operational coach per day

Our results are given in Table 3. As expected, the optimal stock levels were higher for items with higher demand. Furthermore, increasing the penalty cost for operational delay due to item stockout also increased the stock level. On the other hand, a decrease in the probability of item repairability resulted in an increase in the optimal stock level.

Table 3. Results for the four problem instances solved.

Item	Problem Instance							
	1		2		3		4	
	Stock Level	HC+SC \$/yr	Stock Level	HC+SC \$/yr	Stock Level	HC+SC \$/yr	Stock Level	HC+SC \$/yr
1	3	22684.31	4	186404.20	4	24335.46	5	188577.20
2	4	47646.30	5	409873.60	6	50326.94	7	412850.60
3	5	176501.00	8	1216088.00	9	199393.40	11	1257535.00
		246831.61		1812365.80		274055.80		1858962.80

The model is separable with respect to the items, which allows solving the model one item at a time. The expected annual holding cost is a decreasing function with respect to the stock level, S, while the expected annual shortage cost function increases with respect to "S." It follows, then, that the sum of the expected annual holding and shortage costs is a convex function. This property implies that a unique optimal solution exists for each item.

Conclusion

We have presented a model for finding the optimal stock levels of repairable critical items, taken in the context of a rail transport system. The items are typically characterized by infrequent demand and high cost. The model strives to minimize the sum of the expected annual inventory holding cost and the expected annual cost of operational delays due to item stockouts. The model includes a requirement for minimum item availability on demand. We assume a Poisson demand process for our formulation.

We solved the model for an illustrative problem using hypothetical data to cover different item characteristics.

Taking advantage of the separability of the model relative to the items, and the convexity of the formulation for each item, we solved the problem one item at a time, using a spreadsheet based solver developed for this purpose.

Our formulation has general characteristics which extends its applicability to single echelon repairable inventory systems in general.

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