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A review of recent literature on repairable-item inventory systems

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This review includes papers found in the literature for repairable item inventory systems since 1997. The idea is to complement related earlier reviews until 1997, which are mentioned in the paper for the benefit of the interested reader. The papers are classified into two groups, based broadly on the types of inventory systems: single echelon and multi-echelon systems. A number of modeling approaches found in the papers in both groups are noted.

Keywords**:** repairable inventories, single echelon system, multi-echelon system

Introduction

Repairable items are items which, upon failure, are capable of being restored back to working condition through repair. These items are usually high valued, infrequently demanded, and highly essential to operations. Because of their high cost, these items are generally cheaper to repair than to replace.

Although comprising a small percentage of the number of items stocked, repairable items usually account for a significant part of investment in inventories. In Nahmias [1], it is reported that repairable items represented approximately $10 billion in the US military sector alone in 1976. This amount went up to $34 billion in 1994, as noted by Diaz and Fu [2] based on a 1994 private communication from O'Malley. By 2000, the US military had $46.4 billion of repairable-item inventory, as noted by Jung *et al*. [3], based on the US Defense Department supply system inventory report of September 2000. Meanwhile, Diaz and Fu [2] observe that repairable items represent almost 2/3 of the total inventory value, both in the Caracas subway station in Venezuela and in the US Air Force.

The management of repairable inventories has received considerable research attention since the 1960s. Early papers were concerned with items in the military sector. Throughout the years, studies in this area have dealt with items in the aviation industry, e.g., aircraft and its components; transportation industry, e.g., trains, buses and their components; and certain commercial products such as computers and copy machines. The present paper surveys the literature on repairable-item inventory management from 1997 onwards, thus complementing earlier reviews by Nahmias [1], Mabini and Gelders [4], Diaz and Fu [2], and Guide and Srivastava [5].

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In Nahmias [1], we find a survey of mathematical models for determining stocking levels of repairable inventories. The models are classified into three groups, namely, continuous review, periodic review, and models based on cyclic queueing systems. The analytical approaches employed in developing and solving the models are outlined in the paper. Meanwhile, the article of Mabini and Gelders [4] is a partial review of the printed literature on repairable item inventory systems

Diaz and Fu [2] focus on multi-echelon models for repairable inventories, highlighting certain key relationships. They review Sherbrooke's Multi-Echelon Technique for Recoverable Item Control (METRIC) model [6], its extensions and variations, and discuss a number of general queueing models. Finally, they take up the elements in the models that must be addressed in order for them to be applicable to industrial situations.

Guide and Srivastava [5] examine various repairable inventory models proposed in the literature, in the light of the major underlying assumptions for those models. The authors use a three-way classification for the models: solution methodology, single versus multi-echelon, and exact versus approximate solutions.

In this paper, we present the material in repairable-item inventory literature since 1997, in two general groups: single echelon and multi-echelon inventory systems. The modelling approaches used in the papers in both groups fall in one or a combination of the following classifications: (1) continuous review, (2) periodic review, (3) queueing-based models, (4) Markov models, (5) heuristics, and (6) simulation. It is worthy to mention that, where work prior to 1997 is relevant to models included in the present survey, such work is given reference here.

Single-echelon systems

A single-echelon repairable inventory system usually consists of a repair shop and a warehouse which holds inventories of serviceable items and non-serviceable ones awaiting repair. The system has two possible sources of serviceable inventories, namely, returns from repair and external purchases.

The single-echelon problem is usually modelled either as a deterministic problem with batch procurement and repair, or as a stochastic problem with one-for-one replenishment. The studies of Teunter [7, 8], Koh *et al.* [9], and Konstantaras [10] belong to the former group. Teunter [7] proposes EOQ formulas for the manufacturing and recovery batch quantities of items that can be recovered, i.e., repaired, refurbished, or remanufactured. His model allows disposal, and differentiates holding costs for manufactured and recovered items. In another paper, Teunter [8] investigates inventory systems in situations where demand may be filled with new or recovered items, where recovered items are considered to be "as good as new." Assuming deterministic demand rate and return fraction, he presents formulas for determining optimal lot sizes for the production or purchase of new items and for the recovery of returned items. The objective is to minimize the total of lot ordering costs for production and for recovery, and inventory holding costs for recoverable items and new/recovered items. The formulas derived are valid for finite and infinite production and recovery rates.

The contribution of Koh *et al.* [9] is a joint EOQ and EPQ model for the situation where stationary demand can be met from recycles and newly purchased products. The assumption is that a given proportion of used products are collected from customers and subsequently recovered to "good as new" condition. The model finds the EOQ for newly purchased products and the optimal inventory level of recoverable items that would initiate the recovery process. Extending the work of Koh *et al.* [9], Konstantaras [10] presents a formula for determining the optimal inventory level of recoverable items that would mark the start of the inspection and recovery processes, and the economic order quantity for procurement. In this study, Konstantaras considers that some recovered items are perceived to be of secondary quality and can be sold at a reduced price.

A related work by Omar and Yeo [11] introduces an inventory model for the production of new items and repair of used ones. The model assumes a known and finite planning horizon, deterministic demand, and constant repair rates. A numerical solution procedure is proposed and numerical examples are provided in the paper.

Stochastic models for the single-echelon problem are found in Zijm and Avsar [12], Wong *et al.* [13], Wong *et al.* [14], Wong *et al.* [15], Chakravarty [16], Mirzahosseinian [17], Selçuk [18], and Tracht [19], among others. Zijm and Avsar [12] study a two-indenture maintenance system for a number of identical installations which are in use at a single site. The installations are assemblies composed of repairable components. Inventories of ready-for-use assemblies and components are maintained at particular stock points, each one employing a base-stock policy. Allowing backorders and assuming that an assembly failure is caused by only one component at a time, the paper proposes first a slightly aggregated (but exact) formulation for the system, and subsequently presents a near-product-form solution as an approximate steady-state distribution of the aggregated system. The approximate performance measures are found to be accurate by simulation, and are subsequently used to find the optimal base-stock levels, following a greedy procedure.

A number of papers in this group consider pooling and sharing of stocks to meet demand. Wong *et al.* [13] consider a single-item repairable inventory system involving multiple hubs and multiple companies. They present a model for finding stocking levels of spare parts for the system where complete pooling of stock is allowed among the hubs and companies. The model minimizes the total of holding, downtime, and transshipment cost, for which the authors propose a two-stage solution. In Wong *et al.* [14], non-zero lateral transshipment time and delayed lateral transhipments are considered for estimating several performance measures in a single-item repairable inventory system involving multiple companies. Allowing complete pooling of stock among the companies, the authors present a multi-dimensional Markovian model for the problem, and a two-stage solution method for the same.

The multi-item case is studied by Wong *et al.* [15] for a two-location repairable spare parts inventory system, which supports expensive technical systems with high target availability levels. A continuous review model is proposed for the inventory system, where lateral and emergency shipments are allowed to absorb stockout situations. The model aims to minimize the total cost inclusive of inventory holding, lateral transhipments and emergency shipments, subject to an average waiting time limitation per demanded part at each of the two locations. The authors propose a solution procedure based on Lagrangian relaxation to find upper and lower bounds on the optimal total cost; they eventually show a narrow average gap between these bounds.

The paper of Kilpi [20] specifies three cooperative strategies for making repairable aircraft components available to participating airlines when needed. Using simulation, the strategies ad hoc cooperation, cooperative pooling and commercial pooling are studied and compared to the alternative of acting alone, i.e. solo strategy. Findings indicate that generally, the benefits from pooling are greater when demand for one component type served by one pool is higher. The author notes, however, that conflicting interests can complicate the development of efficient pools.

Several studies on the single-echelon system pay special attention to the repair aspect. In his paper, De Haas [21] uses service level as a measure of performance for a maintenance support system consisting of a group of repair departments and stock locations intended to provide serviceable engines to aircraft. The author shows that the service level targets for the repair departments can be significantly lower than those set for the stock locations of the system. The importance of coordinating the performance of the respective repair departments and the stock locations is emphasized. The paper shows that the well-known METRIC [6] and MOD-METRIC [22] models can be used to support the deduction of the service level targets.

Chakravarty [16] studies a reliability system with identical, repairable components, and at least k out of N components are needed for the system to operate. Failed components are attended to by a single repairman on a first-come-first-served basis. The system has K spares which can be tapped to extend its lifetime, following a probabilistic rule. The delivery time of a spare is exponentially distributed; and multiple requests for spares at any given time are possible. The influence of delivery times on the performance measures of the system is investigated, using a finite quasi-birth-and-death process to analyze the system.

Mirzahosseinian [17] investigates a repairable parts system operating under a Performance-Based Logistics (PBL) contract. The closed-loop inventory system is modeled as an M/M/m queue with Poisson failures for components and exponential repair times. Analysis of key model parameters indicated that system availability can be improved by improving the component reliability and the efficiency of the repair facility, rather than improving the base stock level. The latter is said to have little effect on system availability.

Alfredsson [23] presents a mathematical framework for finding the quantity of spares and test equipment needed and for determining where repair should take place in a support system for a fleet of technical systems. The author proposes an algorithm to find cost-efficient configurations for the support system. Tiemessen [24] tackles the repair scheduling problem for a system consisting of a repair shop and one stockpoint. The latter is where spare parts of multiple critical repairable items are kept on stock to support technical equipment. For the objective of minimizing aggregate downtime, with backorders allowed, the study evaluates a number of dynamic scheduling policies, including the myopic allocation rule from the make-to-stock environment. This rule selects the SKU with the largest reduction in expected backorders per invested time unit. The myopic allocation rule is shown to outperform the other heuristic scheduling rules investigated.

While many studies assume ample repair capacity, this assumption may not be realistic in many situations. Some authors have, therefore, investigated the repairable inventory problem with repair capacity limitations. Considering a single-item, single-location problem with limited repair capacity, Selçuk [18] proposes an adaptive base stock policy in which the base stock level of a repairable item is updated based on the work-in-process inventory level in the repair facility. The update frequency is modeled as a separate control parameter along with a standard base stock level. Emergency shipments are employed in stockout situations, whereas priority shipments are used when updating the base stock level. The problem is modelled as a 2-dimensional continuous-time Markov chain, and solved by matrix geometric methods. Numerical results show that, for a given downtime target, inventory on hand and total cost can substantially be reduced with the new policy.

Meanwhile, the work of Tracht [19] applies to the repairable item system for expensive parts of machines. Here, the impact of varying repair capacity on the system is investigated. The study shows how the stock levels should be adjusted so that a maximum backorder level of waiting request is guaranteed for the entire year.

Multi-echelon systems

A multi-echelon inventory system usually refers to a two-level system composed of a number of locations, called bases, supported by a central depot. Items fail at the base level, and are repaired there, if possible. If a base has no repair capacity, failed items are sent to the depot for repair and for replacement. The pioneering work of Sherbrooke (1968), known as METRIC, or Multi-Echelon Review Technique for Repairable Item Control, is the forerunner of studies on multi-echelon repairable inventory systems.

Deterministic models for the multi-echelon repairable inventory system are introduced in Toptal [25] and in Perlman and Levner [26]. Toptal [25] considers a system consisting of a central depot where failed items are recovered, and a collection center where failed items from retailers are temporarily stored until they are brought to the central depot for recovery. A mathematical model is proposed for determining the economic shipment quantities of failed and recovered items between the depot and the collection center. The model strives to minimize the long-run average total cost subject to a service level constraint. The latter is given by the maximum level of failed items in each cycle which are not immediately replaced.

Perlman and Levner [26] propose a network model for an inventory system which covers repairable equipment located at several operational sites in different areas. The multiple supplier inventory system has an internal repair shop offering several modes of repair with different repair times. Spare parts come from an external supplier. The network model, which considers backorders, is shown to efficiently solve the problem for deterministic demands that vary over time.

Most other multi-echelon models are stochastic, and follow a one-for-one replenishment policy. Kim *et al.* [27] address the problem of finding the initial spare inventory level for a multi-echelon repairable item inventory system, with a general repair time distribution. He presents an algorithm for determining the inventory level which minimizes the total expected cost while satisfying a minimum service rate.

The model by Tracht [28] determines cost-optimal inventory levels for warehouses in a two-echelon spare parts supply chain in the aviation industry. The method takes into account budget and inventory level limitations. A simulation model is used to validate the calculated inventory levels. Costantino [29] presents a spare parts allocation model for the Italian Air Force, whose objective is to minimize backorders and ensure a 99% system availability. Solved by marginal analysis, the model incorporates repair centers with different skills in a multi-echelon, multi-item system, where items have a multi-indentured structure.

A number of authors have also considered the multi-echelon problem with repair capacity limitations: Díaz and Fu [30], Perlman *et al.* [31], Sleptchenko *et al.* [32], Jung *et al.* [3], Spanjers [33], Lau and Song [34], and Tao [36]. Díaz and Fu [30] study a system with limited repair facilities at the depot, where all failed LRUs are repaired. They propose approximations based on queuing theory for three cases of queueing at the depot: *M/M/s* single-class model, *M/G/s* single-class model and *M/G/s* multi-class model. Meanwhile, the work of Perlman, *et al.* [31]

describes three models to investigate the effect of congestion in a multi-echelon inventory system employing two modes of repair, each with limited repair capacity. An expanding repair policy employed at the bases for choosing which mode of repair to use is compared with various expediting policies related to congestion externalities. The expanding repair policy which considers congestion externalities is shown to result in better system performance measurement than an expanding policy with no congestion.

Meanwhile, Sleptchenko *et al.* [32] investigate the capacitated multi-echelon problem for repairable service parts with a hierarchical (multi-indentured) structure. They show that if the repair shop utilization is relatively high, the frequently used infinite capacity assumption may affect system performance and stock allocation decisions badly. The paper presents a modified VARI-METRIC [35] method to allocate service part stocks in the network for both cases of item-dedicated and shared repair shops. The repair shops are modelled as multi-server queueing systems. Validated by simulation, the technique is found to be more accurate than the VARI-METRIC technique.

The model by Jung *et al.* [3], developed in the context of a system with finite repair channels, allows the lateral transshipment of a spare from another base, when needed and where possible, based on the availability of stocks there. If no such spare is available from any of the bases, the item is backordered. The authors propose an algorithm to find the spare inventory level at each base, which minimizes the total expected cost.

The work of Spanjers [33] presents a slightly aggregated model for finding the steady state probabilities for a closed loop two-echelon system with finite repair capacities at the bases and at the depot. Ready-for-use machines are held in stock at these locations; and backlogging is allowed on both levels. A near-product-form solution is proposed for determining excellent approximations of relevant performance measures. These approximations are then used to find stock levels which maximize system availability subject to a budget constraint.

Lau and Song [34] address the corrective maintenance problem in military logistics and present an analytical model for evaluating system performance under limited repair capacity and nonstationary demands. They develop a METRIC-based optimization algorithm to solve the model. In Tao [36], we find a discrete-event stochastic simulation model for a closed loop multi-echelon repairable inventory system. Using examples, the paper shows how the model can be used to study the effect of depot repair capacity on expected backorders and to determine spares allocation. The results indicate that, compared to METRIC, the model is more accurate and efficient.

Conclusion

The management of repairable inventories has become increasingly important is the last few decades, primarily because of the significant share of repairable items in the inventory investment of an organization. This paper is aimed to help present and future researchers in this area, in terms of assessing the work already done and determining which important issues remain unresolved. It is hoped that this paper will be a useful guide to the researcher on repairable inventories in determining future research directions in this area.

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